Part A

1. B
2. C
3. C
4. B
5. D
6. A
7. D
8. C
9. C
10. A
11. B
12. D
13. C
14. C
15. C
16. D
17. B
18. B
19. D
20. D
21. C
22. C
23. B
24. C
25. B
26. B
27. D
28. C
29. C
30. A

Part B

31. D
32. A
33. B
34. C
35. D
36. A
37. B
38. B
39. C
40. D
41. B
42. C
43. D
44. B
45. C

CALCULUS AB

SECTION I, Part A Solutions

1. Correct answer: (B)

To find the slope, we need to calculate $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$\frac{dy}{dy} = 6$ and $\frac{dx}{dt} = -4$

Therefore,

$$\frac{dy}{dx} = \frac{6}{-4} = -\frac{3}{2}$$

2. Correct answer: (C)

Apply L'Hospital's Rule.

$$\lim_{x \to 1} \frac{\int_1^x e^t - 1 \ dt}{x^2 - 1} = \lim_{x \to 1} \frac{e^x - 1}{2x} = \frac{e - 1}{2}$$

3. Correct answer: (C)

$f(x) = k$ for $x = 4$, so $f(4) = k$.

To find the value of k where f is continuous at $x=4$, we need to find $\lim_{x\to 4} f(x) = f(4)$.

$$f(4) = k = \lim_{x\to 4}\frac{\sqrt{3x+4}-\sqrt{2x+8}}{x-4} = \lim_{x\to 4}\frac{\sqrt{3x+4}-\sqrt{2x+8}}{x-4}\cdot\frac{\sqrt{3x+4}+\sqrt{2x+8}}{\sqrt{3x+4}+\sqrt{2x+8}}$$

$$= \lim_{x\to 4}\frac{(3x+4)-(2x+8)}{(x-4)(\sqrt{3x+4}+\sqrt{2x+8})} = \lim_{x\to 4}\frac{x-4}{(x-4)(\sqrt{3x+4}+\sqrt{2x+8})}$$

$$= \lim_{x\to 4}\frac{1}{\sqrt{3x+4}+\sqrt{2x+8}} = \frac{1}{\sqrt{16}+\sqrt{16}} = \frac{1}{4+4} = \frac{1}{8}$$

4. Correct answer: (B)

The fourth-degree Taylor polynomial for $\ln(1+x)$ about $x=0$ is

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$$

Then

$$\ln 2 = \ln(1+1) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$$

5. Correct answer: (D)

This is a geometric series with $r = \frac{x-2}{5}$. The geometric series convergence for $-1 < r < 1$.

So we have

$$-1 < \frac{x-2}{5} < 1$$

$$-5 < x - 2 < 5$$

$$-5 + 2 < x < 5 + 2$$

$$-3 < x < 7$$

6. Correct answer: (A)

To find the length of the parametric curve, we use

$$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt$$

First, find $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

$$\frac{dx}{dt} = 16t^7$$

$$\frac{dy}{dt} = 16t^3$$

Therefore,

$$\int_0^1 \sqrt{(16t^7)^2 + (16t^3)^2}\ dt = \int_0^1 \sqrt{16^2(t^{14} + t^6)}\ dt$$

$$= 16\int_0^1 \sqrt{t^{14} + t^6}\ dt$$

7. Correct answer: (D)

Use the chain rule.

$$y' = 4(\sqrt{x} + \sin x)^3 \cdot \frac{d}{dx}(\sqrt{x} + \sin x) = 4(\sqrt{x} + \sin x)^3\left(\frac{1}{2\sqrt{x}} + \cos x\right)$$

8. Correct answer: (C)

Use the quotient rule.

$$f'(x) = \frac{\frac{d}{dx}(x) \cdot \tan x - x \cdot \frac{d}{dx}(\tan x)}{(\tan x)^2} = \frac{1 \cdot \tan x - x \sec^2 x}{\tan^2 x} = \frac{\tan x - x \sec^2 x}{\tan^2 x}$$

$$= \frac{1}{\tan x} - \frac{x \sec^2 x}{\tan^2 x} = \cot x - x \cdot \frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} = \cot x - \frac{x}{\sin^2 x} = \cot x - x \csc^2 x$$

Then

$$f'\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}} - \frac{\pi}{3}\left(\frac{2}{\sqrt{3}}\right)^2 = \frac{1}{\sqrt{3}} - \frac{4\pi}{9} = \frac{3\sqrt{3} - 4\pi}{9}$$

9. Correct answer: (C)

The stem of the question means $f'(3) = 9$. So f is differentiable at $x = 3$ and therefore continuous at $x = 3$. We know nothing about the continuity of f'. Therefore, the only correct statements are I and II.

10. Correct answer: (A)

The average value of the function $f(x)$ on the interval $[a, b]$ is

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\ dx$$

$$f_{ave} = \frac{1}{2-0}\int_0^2 x^2\sqrt{3x^3+1}\ dx$$

Let $u = 3x^3 + 1$, $du = 9x^2\ dx$, and $\frac{du}{9} = x^2\ dx$.

$$f_{ave} = \frac{1}{2}\int_0^2 \frac{1}{9}\sqrt{u}\ du = \frac{1}{9 \cdot 2}\int_0^2 \sqrt{u}\ du = \frac{1}{18} \cdot \frac{2}{3}u^{\frac{3}{2}}\Big|_0^2 = \frac{1}{27}(3x^3+1)^{\frac{3}{2}}\Big|_0^2$$

$$= \frac{1}{27}((3(2)^3+1)^{\frac{3}{2}} - (3(0)^3+1)^{\frac{3}{2}}) = \frac{1}{27}((25)^{\frac{3}{2}} - (1)^{\frac{3}{2}}) = \frac{1}{27}(125-1) = \frac{124}{27}$$

11. Correct answer: (B)

To find when a function is decreasing, you must first take the derivative, then set it equal to 0, and then find between which zero values the function is negative.

$$f'(x) = 3x^2 - 75$$

$$3x^2 - 75 = 0$$

$$3(x^2 - 25) = 0$$

$$(x-5)(x+5) = 0$$

$$x = \pm 5$$

Now test values on all sides of these to find when the function is negative, and therefore decreasing. You could pick -6, 0, and 6.

$$f'(-6) = 3(-6)^2 - 75 = 3 \cdot 36 - 75 = 108 - 75 = 33$$

$$f'(0) = 3(0) - 75 = -75$$

$$f'(6) = 3(6)^2 - 75 = 3 \cdot 36 - 75 = 108 - 75 = 33$$

Since the values that are negative occur when $x = 0$, the function is decreasing on the intervals that include these values, so the function is decreasing on $[-5,5]$.

12. Correct answer: (D)

To find this integral, we need to use integration by parts. Let $u = \ln 2x$ and $du = \frac{1}{x}\ dx$, $dv = 3x^2$ and $v = \int 3x^2\ dx = x^3$.

Substituting into the integration by parts formula gives

$$\int u\ dv = uv - \int v\ du$$

$$\int 3x^2 \ln 2x\ dx = \ln 2x \cdot x^3 - \int x^3 \cdot \frac{1}{x}\ dx$$

$$= x^3 \ln 2x - \int x^2\ dx$$

$$= x^3 \ln 2x - \frac{x^3}{3} + C$$

13. Correct answer: (C)

Remember that

$$v(t) = s'(t)$$

So

$$v(t) = \left(\frac{3t^2 + 2t}{t^3 + t^2}, 15t^2 \right) = \left(\frac{3t + 2}{t^2 + t}, 15t^2 \right)$$

$$a(t) = v'(t) = \left(\frac{3(t^2 + t) - (2t + 1)(3t + 2)}{(t^2 + t)^2}, 30t \right)$$

Then

$$a(1) = \left(\frac{3(1 + 1) - (2 + 1)(3 + 2)}{(1 + 1)^2}, 30 \right) = \left(-\frac{9}{4}, 30 \right)$$

14. Correct answer: (C)

Right-hand sum: $f(t_1)\Delta t + f(t_2)\Delta t + f(t_3)\Delta t + f(t_4)\Delta t$.

Substitute to get

$$5.0(8 - 5) + 4.0(11 - 8) + 3.5(15 - 11) + 2.0(19 - 15)$$

$$= 5(3) + 4(3) + 3.5(4) + 2(4) = 15 + 12 + 14 + 8 = 49$$

The approximation of the number of liters of water that are in the tank at time $t = 19$ hours is $30 + 49 = 79$ liters.

15. Correct answer: (C)

First, find the points of intersection of the two curves to determine the limits of integration.

$1 + \sin\theta = 1$

$\sin\theta = 0$

$\theta = \pi$ and $\theta = 2\pi$

Therefore, the area is

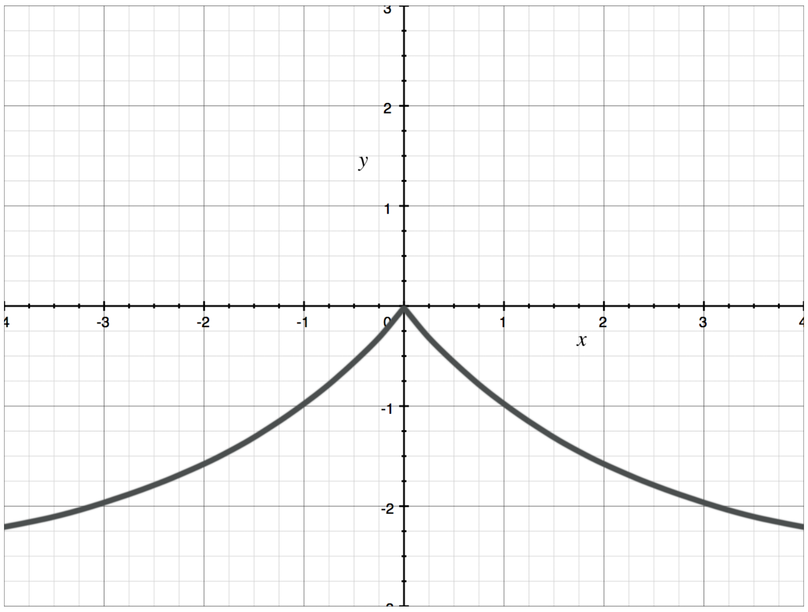
$$A = \pi r^2 - \frac{1}{2}\int_{\pi}^{2\pi} (1^2 - (1 + \sin\theta)^2)\ d\theta$$

$$= \pi - \frac{1}{2}\int_{\pi}^{2\pi} 1 - (1 + 2\sin\theta + \sin^2\theta)\ d\theta$$

$$= \pi - \frac{1}{2}\int_{\pi}^{2\pi} -2\sin\theta - \sin^2\theta\ d\theta$$

$$= \pi + \int_{\pi}^{2\pi} \sin\theta\ d\theta + \frac{1}{2}\int_{\pi}^{2\pi} \sin^2\theta\ d\theta = \pi + \frac{1}{2}\int_{\pi}^{2\pi} 2\sin\theta + \sin^2\theta\ d\theta$$

16. Correct answer: (D)



f is continuous at $x = 0$.

$$f'(x) = -\frac{1}{2\sqrt{|x|}}$$

Therefore, f is not differentiable at $x = 0$.

17. Correct answer: (B)

The Taylor series for e^x is

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

Then

$$e^{-2x} = 1 + (-2x) + \frac{(-2x)^2}{2!} + \frac{(-2x)^3}{3!} + \dots$$

$$= 1 - 2x + \frac{4x^2}{2} - \frac{8x^3}{6} + \dots$$

$$= 1 - 2x + 2x^2 - \frac{4}{3}x^3 + \dots$$

Therefore, the coefficient on x^3 is $-\frac{4}{3}$.

18. Correct answer: (B)

$$\int_0^3 f(x)\ dx = \int_0^1 f(x)\ dx + \int_1^3 f(x)\ dx$$

$$= \int_0^1 (2x+4)\ dx + \int_1^3 (-3x+9)\ dx$$

$$= x^2 + 4x\Big|_0^1 + \left[-\frac{3x^2}{2} + 9x\right]_1^3$$

$$= (1+4-0) + \left(-\frac{27}{2} + 27 - \left(-\frac{3}{2} + 9\right)\right)$$

$$= 5 + \frac{27}{2} - \frac{15}{2} = 5 + 6 = 11$$

19. Correct answer: (D)

First, find $f(g(x))$. Substitute $g(x)$ for x into $f(x)$.

$$f(g(x)) = 2(2x+3)^3$$

Then

$$\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx}f(g(x)) = 2 \cdot 3(2x+3)^2 \cdot 2 = 12(2x+3)^2$$

Now evaluate the derivative at $x = 1$.

$$\frac{d}{dx}(f(g(1))) = 12(2(1) + 3)^2 = 12(5)^2 = 12 \cdot 25 = 300$$

20. Correct answer: (D)

From the graph we see that $g(5) = \int_0^5 f(t)\ dt$ is positive.

$$g'(x) = \frac{d}{dx}\int_0^x f(t)\ dt = f(x)$$

Therefore

$$g'(5) = f(5) = 0$$

$$g'(x) = f(x)$$

$$g''(x) = f'(x)$$

So $g''(5) = f'(5)$, which is the slope at $x = 5$. From the graph we see that the slope at $x = 5$ is negative. Therefore, $g''(5) < g'(5) < g(5)$.

21. Correct answer: (C)

At rest, $v(t) = 0$. Also, $v(t) = x'(t)$.

Use the product rule.

$$v(t) = 2te^{-3t} - 3t^2e^{-3t}$$

Now solve $v(t) = 0$.

$$2te^{-3t} - 3t^2e^{-3t} = 0$$

$$e^{-3t}(2t - 3t^2) = 0$$

$$e^{-3t}(t)(2 - 3t) = 0$$

$$t = 0, \frac{2}{3}$$

22. Correct answer: (C)

$$\frac{d}{dx}\int_0^x g(t)\ dt = g(x)$$

Therefore

$$\frac{d}{dx}(f(x)) = f'(x) = g(x)$$

The graph shows that f is decreasing on the interval $(-4,0)$ and increasing on the interval $(0,4)$. This means the graph of the derivative of f is negative on the interval $(-4,0)$ and positive on the interval $(0,4)$.

23. Correct answer: (B)

The slope of the tangent line is $\frac{1}{2}$, which means that $f'(x) = \frac{1}{2}$.

Find the derivative using the quotient rule.

$$f'(x) = \frac{2x(2x+1) - x^2(2)}{(2x+1)^2} = \frac{4x^2 + 2x - 2x^2}{(2x+1)^2} = \frac{2x^2 + 2x}{(2x+1)^2}$$

Solve the equation.

$$\frac{2x^2+2x}{(2x+1)^2}=\frac{1}{2}$$

$$2(2x^2+2x)=(2x+1)^2$$

$$4x^2+4x=4x^2+2x+1$$

$$2x=1$$

$$x=\frac{1}{2}$$

Then $f\left(\frac{1}{2}\right)=\frac{\left(\frac{1}{2}\right)^2}{2\cdot\left(\frac{1}{2}\right)+1}=\frac{\frac{1}{4}}{2}=\frac{1}{8}$.

24. Correct answer: (C)

Since g is the inverse function of f, if $f(1)=0$, then $g(0)=1$.

Remember that if $f^{-1}(x)=g(x)$, then $g'(x)=\frac{1}{f'(g(x))}$. Therefore,

$$g'(0)=\frac{1}{f'(g(0))}=\frac{1}{f'(1)}$$

Now find the derivative of $f(x)$ at $x=1$.

$$f'(x)=5x^4$$

$$f'(1)=5\cdot(1)^4=5$$

So $g'(0)=\frac{1}{5}$.

25. Correct answer: (B)

To find the vertical asymptotes, set the denominator equal to 0. Therefore, we have

$$c - x^2 = 0$$

$$c - 2^2 = 0$$

$$c - 4 = 0$$

$$c = 4$$

For $x = -2$ we also get $c = 4$.

If the numerator and denominator are of equal degree, then the ratio of coefficients will give the horizontal asymptote. In our case, we have ax^2 in the numerator and $-x^2$ in the denominator. Therefore, $\frac{ax^2}{-x^2} = -a = 5$, so $a = -5$. Now $a + c = -5 + 4 = -1$.

26. Correct answer: (B)

Start by finding the derivative of the function.

$$f'(x) = e^{2x} + 2xe^{2x}$$

At critical points, $f'(x) = 0$.

$$e^{2x} + 2xe^{2x} = 0$$

$$e^{2x}(1 + 2x) = 0$$

$$1+2x=0$$

$$x=-\frac{1}{2}$$

$$f\left(-\frac{1}{2}\right)=\left(-\frac{1}{2}\right)e^{2\left(-\frac{1}{2}\right)}=-\frac{1}{2e}$$

The absolute minimum value is $-\frac{1}{2e}$.

27. Correct answer: (D)

Separate variables.

$$\frac{dy}{dt}=k$$

$$dy=k\ dt$$

Integrate both sides.

$$y=kt+c$$

Use the values in the table.

$$3=k(0)+c$$

$$c=3$$

$$7=k(1)+3$$

$$k=4$$

Therefore, we get $y=4t+3$.

28. Correct answer: (C)

Start by finding derivative of the function.

$$f'(x) = 2xe^{-kx} + (x^2+1)(-ke^{-kx}) = e^{-kx}(2x - kx^2 - k)$$

At critical point $f'(x) = 0$, DNE, we get:

$$e^{-kx}(2x - kx^2 - k) = 0$$

$$e^{-kx} \neq 0, \text{ so } 2x - kx^2 - k = 0$$

We know that critical point is at $x = 1$. Substitute and find the value of k.

$$2(1) - k(1)^2 - k = 0$$

$$2 - k - k = 0$$

$$2 = 2k$$

$$k = 1$$

29. Correct answer: (C)

First separate the variables. Multiply both sides by dx and divide both sides by e^{-y}.

$$\frac{dy}{dx} = xe^{-y}$$

$$\frac{dy}{e^{-y}} = x\ dx$$

$$e^y\ dy = x\ dx$$

Integrate both sides.

$$\int e^y\ dy = \int x\ dx$$

$$e^y + c_1 = \frac{x^2}{2} + c_2$$

$$e^y = \frac{x^2}{2} + c, \text{ where } c = c_2 - c_1$$

$$\ln e^y = \ln\left(\frac{x^2}{2} + c\right)$$

$$y = \ln\left(\frac{x^2}{2} + c\right)$$

Since $y(0) = 1$, substitute and find c.

$$0 = \ln\left(\frac{1}{2} + c\right)$$

We remember that $\ln 1 = 0$, so $\frac{1}{2} + c = 1$, and $c = \frac{1}{2}$.

Therefore, we get $y = \ln\left(\frac{x^2}{2} + \frac{1}{2}\right)$.

30. Correct answer: (A)

Since the position is $x(t) = \cos t + \sin t$, the velocity is $v(t) = x'(t) = -\sin t + \cos t$.

We first need to find the point where velocity is first equal to 0.

$$-\sin t + \cos t = 0$$

$\cos t = \sin t$, which is true when $t = \frac{\pi}{4}$.

Now we know that

$$a(t) = v'(t) = -\cos t - \sin t$$

and

$$a\left(\frac{\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)$$

$$= -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$$

CALCULUS AB

SECTION I, Part B Solutions

31. Correct answer: (D)

We can rewrite the equation as $\frac{dy}{dx}=\frac{2y-2xy^3}{3x^2y^2+2x}$.

Now find $\frac{dy}{dx}$ at the point $(-1,1)$.

$$\frac{dy}{dx}(-1,1)=\frac{2(1)-2(-1)(1)^3}{3(-1)^2(1)^2+2(-1)}=\frac{2+2}{3-2}=\frac{4}{1}=4$$

Then use the quotient rule.

$$\frac{d^2y}{dx^2}=\frac{(2y-2xy^3)'(3x^2y^2+2x)-(3x^2y^2+2x)'(2y-2xy^3)}{(3x^2y^2+2x)^2}$$

$$=\frac{\left(2\frac{dy}{dx}-2y^3-6xy^2\frac{dy}{dx}\right)(3x^2y^2+2x)-\left(6xy^2+6x^2y\frac{dy}{dx}+2\right)(2y-2xy^3)}{(3x^2y^2+2x)^2}$$

$$=\frac{\left(2(4)-2(1)-6(-1)(1)(4)\right)\left(3(1)(1)+2(-1)\right)-\left(6(-1)(1)+6(1)(1)(4)+2\right)\left(2(1)-2(-1)(1)\right)}{\left(3(1)(1)+2(-1)\right)^2}$$

$$=\frac{(8-2+24)(1)-(20)(0)}{1}=30$$

32. Correct answer: (A)

We can rewrite $f(t)$ and its first and second derivative as

$$f(t) = (e^{2t}, \sin t)$$

$$f'(t) = (2e^{2t}, \cos t)$$

$$f''(t) = (4e^{2t}, -\sin t)$$

33. Correct answer: (B)

$$1 - 2x + \frac{4x^2}{2!} - \frac{8x^3}{3!} + \ldots + \frac{(-1)^n(2x)^n}{n!} = e^{-2x}$$

Using a calculator, we can find the intersection of the graphs.

$$e^{-2x} = x^2 + 4$$

$$x \approx -0.761$$

34. Correct answer: (C)

Since the graph crosses the x-axis at one point, then the y-value is equal to 0. Substitute $y = 0$ into the equation and find the x-value.

$$0 = 3\ln(\sec x)$$

$\ln 1 = 0$, so $\sec x = 1$ and $x = 2\pi n$, where n is any integer

We have the interval $[6,7]$, so $x = 2\pi \approx 6.2832$.

To find the slope, we need to find y'.

$$y' = 3\frac{\sec x \tan x}{\sec x} = 3\tan x$$

$$y'(2\pi) = y'(6.2832) = 3\tan 2\pi = 0$$

35. Correct answer: (D)

Points of inflection occur where f' changes from increasing to decreasing or from decreasing to increasing. There are six such points. f' changes from positive to negative at $x = 5$, therefore f has a relative maximum at $x = 5$.

Since f' is decreasing on the interval $[3,6]$, the function f is concave down on the interval $[3,6]$. Therefore, only statements I and III are true.

36. Correct answer: (A)

First, we need to rewrite S_n as

$$S_n = \frac{(2+n^2)^{25}}{(3+n^2)^{25}} \cdot \frac{3^{n+1}}{3^{n+2}} = \frac{1}{3}\left(\frac{2+n^2}{3+n^2}\right)^{25}$$

Then

$$\lim_{n\to\infty} \frac{1}{3}\left(\frac{2+n^2}{3+n^2}\right)^{25} = \lim_{n\to\infty} \frac{1}{3}\left(\frac{\frac{2}{n^2}+1}{\frac{3}{n^2}+1}\right)^{25} = \lim_{n\to\infty} \frac{1}{3} \cdot 1^{25} = \frac{1}{3}$$

37. Correct answer: (B)

To find the area, we use the formula

$$A = \int \frac{1}{2}(r_2 - r_1)^2 \ d\theta$$

First, we need to find the point of intersection to be able to find the limits of integration.

$$4\sin\theta = 2$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

Therefore, we get

$$A = \frac{1}{2}\int_0^{\frac{\pi}{6}} (4\sin\theta)^2\ d\theta + \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 2^2\ d\theta + \frac{1}{2}\int_{\frac{5\pi}{6}}^{\pi} (4\sin\theta)^2\ d\theta$$

$$= 8\int_0^{\frac{\pi}{6}} \sin^2\theta\ d\theta + \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 2\ d\theta + 8\int_{\frac{5\pi}{6}}^{\pi} \sin^2\theta\ d\theta$$

We know that $2\sin^2\theta = 1 - \cos 2\theta$, so we get

$$4\int_0^{\frac{\pi}{6}} 1 - \cos 2\theta\ d\theta + \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 2\ d\theta + 4\int_{\frac{5\pi}{6}}^{\pi} 1 - \cos 2\theta\ d\theta$$

$$\approx 0.3623 + 4.1888 + 0.3623 \approx 4.914$$

38. Correct answer: (B)

By ratio test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(x-4)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{(x-4)^n}\right|$$

$$= \lim_{n\to\infty} \left| \frac{(x-4)\sqrt{n}}{\sqrt{n+1}} \right| = |x-4|$$

The power series absolutely converges when

$$|x-4| < 1$$

$$-1 < x-4 < 1$$

$$3 < x < 5$$

When $|x-4| < 1$, we see that it converges when $x=3$ and diverges when $x=5$, because the series will converge if the numerator is $(-1)^n$ and diverge if the numerator is 1^n. Therefore, the power series converges for $3 \le x < 5$.

39. Correct answer: (C)

The area of a triangle is given by the formula $A = \frac{1}{2}bh$. Take the derivative with respect to t of both sides of the equation.

$$\frac{dA}{dt} = \frac{1}{2}\left(h\frac{db}{dt} + b\frac{dh}{dt}\right)$$

Substitute the given rates.

$$\frac{dA}{dt} = \frac{1}{2}(2h+2b) = h+b$$

The area will be decreasing when $\frac{dA}{dt} < 0$, which is true when $h+b<0$, or $h < -b$.

40. Correct answer: (D)

We know that $a(t) = t^2 + \cos t$ and $v(0) = 1$.

$$v(t) = \int a(t)\ dt$$

$$v(t) = \int t^2 + \cos t\ dt = \frac{t^3}{3} + \sin t + C$$

Substitute $v(0) = 1$ and solve for C.

$$0 + \sin 0 + C = -1$$

$$c = -1$$

Therefore

$$v(t) = \frac{t^3}{3} + \sin t - 1$$

Then we need to find t when $v(t) = 0$.

$$\frac{t^3}{3} + \sin t - 1 = 0$$

$$t \approx 0.881$$

41. Correct answer: (B)

Change the integration from x to t.

$t = 3x$, then $dt = 3\ dx$, or $dx = \frac{1}{3}\ dt$.

Use $t = 3x$ to find the new limits of integration.

If $x = 5$, then $t = 3 \cdot 5 = 15$

If $x = 11$, then $t = 3 \cdot 11 = 33$

Then we get

$$\int_5^{11} f(3x)\ dx = \frac{1}{3}\int_{15}^{33} f(t)\ dt$$

$$\frac{1}{3}\int_{15}^{33} f(t)\ dt = 15$$

$$\int_{15}^{33} f(t)\ dt = 15 \cdot 3 = 45$$

42. Correct answer: (C)

Points of inflection occur where f' changes from increasing to decreasing or from decreasing to increasing. According to the table, there are two inflection points.

43. Correct answer: (D)

Since both of the equations are functions, these are upper and lower curves. Find the points of intersection by setting the curves equal to each other.

$$3x^2 + x - 2 = x + 3$$

$$3x^2 - 5 = 0$$

$$x \approx \pm\sqrt{\frac{5}{3}}$$

The function $y = x + 3$ is the upper curve, so the integral to find the area between the two curves is

$$\int_{-\sqrt{\frac{5}{3}}}^{\sqrt{\frac{5}{3}}} (x+3) - (3x^2 + x - 2)\ dx = \int_{-\sqrt{\frac{5}{3}}}^{\sqrt{\frac{5}{3}}} x + 3 - 3x^2 - x + 2\ dx$$

$$\int_{-\sqrt{\frac{5}{3}}}^{\sqrt{\frac{5}{3}}} 5 - 3x^2\ dx = 5x - x^3 \Big|_{-\sqrt{\frac{5}{3}}}^{\sqrt{\frac{5}{3}}}$$

$$5\sqrt{\frac{5}{3}} - \left(\sqrt{\frac{5}{3}}\right)^3 - \left[5\left(-\sqrt{\frac{5}{3}}\right) - \left(-\sqrt{\frac{5}{3}}\right)^3\right]$$

$$5\sqrt{\frac{5}{3}} - \left(\sqrt{\frac{5}{3}}\right)^3 + 5\sqrt{\frac{5}{3}} - \left(\sqrt{\frac{5}{3}}\right)^3 = \frac{20\sqrt{5}}{3\sqrt{3}} \approx 8.607$$

44. Correct answer: (B)

Find the intersection points to find the interval $[a, b]$.

$$x^2 + 3 = x + 3$$

$$x^2 - x + 3 - 3 = x - x + 3 - 3$$

$$x^2 - x = 0$$

$$x(x-1) = 0$$

$$x = 0,\ 1$$

The interval $[a, b]$ is $[0,1]$, $f(x) = x + 3$ because the line is above the parabola in the interval $[0,1]$, and $g(x) = x^2 + 3$.

$$V = \pi \int_a^b [f(x)]^2 - [g(x)]^2\ dx = \pi \int_0^1 (x+3)^2 - (x^2+3)^2\ dx$$

$$= \pi \int_0^1 x^2 + 6x + 9 - (x^4 + 6x^2 + 9)\ dx = \pi \int_0^1 -x^4 - 5x^2 + 6x\ dx$$

$$= \pi \left(-\frac{x^5}{5} - \frac{5}{3}x^3 + 3x^2 \right) \Bigg|_0^1 = \pi \left[-\frac{1^5}{5} - \frac{5}{3}(1)^3 + 3(1)^2 - \left(-\frac{0^5}{5} - \frac{5}{3}(0)^3 + 3(0)^2 \right) \right]$$

$$= \pi \left(-\frac{1}{5} - \frac{5}{3} + 3 - 0 \right) = \pi \left(-\frac{3}{15} - \frac{25}{15} + \frac{45}{15} \right) = \frac{17}{15}\pi \approx 3.560$$

45. Correct answer: (C)

Let x be the distance from the train to the intersection. Then $\frac{dx}{dt} = 65$.

Use the Pythagorean theorem.

$$S^2 = x^2 + 90^2$$

Find the derivative.

$$2S\frac{dS}{dt} = 2x\frac{dx}{dt}, \text{ or } \frac{dS}{dt} = \frac{x}{S}\frac{dx}{dt}$$

After 8 seconds, $x = 65 \cdot 8 = 520$ and so $S = \sqrt{90^2 + 520^2} \approx 527.731$. Therefore,

$$\frac{dS}{dt} = \frac{520}{527.731} \cdot (65) \approx 64.048$$

CALCULUS AB

SECTION II, Part A Solutions

1. Solution:

a. To find how many men enter the cableway during the time interval $0 \leq t \leq 150$, we need to find the integral of $f(t)$ with the limits of integration $0 \leq t \leq 150$.

$$\int_0^{150} \frac{7}{1{,}250}t^2\left(\frac{150-t}{150}\right)^6 \ dt = 75$$

b. Let $M(t)$ be the number of men in line. We know that there are 5 men in line at time $t=0$. Therefore, $M(0)=5$.

$$M(150)-M(0)=\int_0^{150} f(t)-0.4 \ dt$$

$$M(150)=5+\int_0^{150} f(t)-0.4 \ dt=\int_0^{150} \frac{7}{1{,}250}t^2\left(\frac{150-t}{150}\right)^6-0.4 \ dt=20$$

c. Let P be the first time there are no men in line.

$$M(P)-M(150)=\int_{150}^{k} -0.4 \ dt$$

$$0-20=-0.4(k-150)$$

$k=200$ minutes

d. $\frac{dM}{dt}=f(t)-0.4$

Find the time when the number of men in the line for the cableway is at a minimum.

$$f(t) - 0.4 = 0$$

$$\frac{7}{1{,}250}t^2\left(\frac{150-t}{150}\right)^6 - 0.4 = 0$$

$$t_1 \approx 10.5093$$

$$t_2 \approx 78.7026$$

Then

$$M(10.5093) - M(0) = \int_0^{10.5093} f(t) - 0.4\ dt$$

$$M(10.5093) = 5 + \int_0^{10.5093} f(t) - 0.4\ dt$$

$$M(10.5093) \approx 2.3684$$

For $0 \le t \le 150$, the minimum number of men in line is 2 men at $t = 10.5093$ by the extreme value theorem.

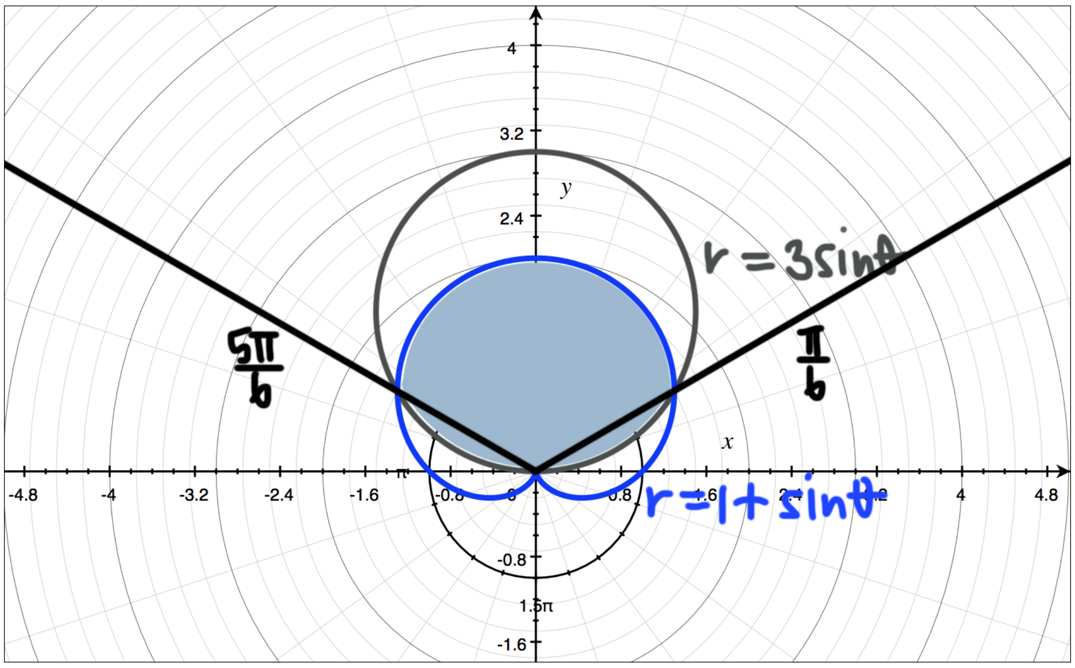
2. Solution:

a. $A = \frac{1}{2}\int_0^{\frac{\pi}{6}} (3\sin\theta)^2\ d\theta + \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (1+\sin\theta)^2\ d\theta + \frac{1}{2}\int_{\frac{5\pi}{6}}^{\pi} (3\sin\theta)^2\ d\theta$

$$= \frac{9}{2}\int_0^{\frac{\pi}{6}} \sin^{\theta}\ d\theta + \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 1 + 2\sin\theta + \sin^2\theta\ d\theta + \frac{9}{2}\int_{\frac{5\pi}{6}}^{\pi} \sin^2\theta\ d\theta$$

Use a calculator.

$$A \approx 0.2038 + 3.5194 + 0.2038 \approx 3.927$$



b. To find the slope of the line tangent to the graph, we need to find $\frac{dy}{dx}$.

We know that

$y = r\sin\theta$ and $x = r\cos\theta$

Then

$$\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$$

$$\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta$$

$$r = 3\sin\theta \to \frac{dr}{d\theta} = 3\cos\theta$$

So

$$\frac{dr}{d\theta}\left(\frac{\pi}{3}\right)=3\cos\frac{\pi}{3}=3\cdot\frac{1}{2}=\frac{3}{2}$$

$$r=3\sin\frac{\pi}{3}=3\cdot\frac{\sqrt{3}}{2}=\frac{3\sqrt{3}}{2}$$

Then

$$\frac{dy}{dx}\left(\frac{\pi}{3}\right)=\frac{\frac{3}{2}\sin\frac{\pi}{3}+\frac{3\sqrt{3}}{2}\cos\frac{\pi}{3}}{\frac{3}{2}\cos\frac{\pi}{3}-\frac{3\sqrt{3}}{2}\sin\frac{\pi}{3}}\approx-1.732$$

c. $x=r\cos\theta$ and $y=r\sin\theta$

Since $r=1+\sin\theta$, we get

$$x=(1+\sin\theta)\cos\theta$$

$$y=(1+\sin\theta)\sin\theta$$

Since we have $\theta=t^2$, then

$$x(t)=(1+\sin t^2)\cos t^2$$

$$y(t)=(1+\sin t^2)\sin t^2$$

$$s(t)=\langle x(t),y(t)\rangle=\langle(1+\sin t^2)\cos t^2,\ (1+\sin t^2)\sin t^2\rangle$$

$$v(t)=s'(t)=\langle x'(t),y'(t)\rangle$$

$$=\langle 2t(\cos^2 t^2-\sin t^2-\sin^2 t^2),\ 4t\sin t^2\cos t^2+2t\cos t^2\rangle$$

$$v(2)=\langle x'(t),\ y'(2)\rangle\approx\langle 2.443,\ 1.343\rangle$$

CALCULUS AB

SECTION II, Part B Solutions

3. Solution:

a. $f'(x) = g(x)$

$$f(x) = \int g(x)\ dx$$

$$f(7) - f(-1) = \int_{-1}^{7} g(x)\ dx$$

$$f(7) = f(-1) + \int_{-1}^{7} g(x)\ dx$$

$$f(7) = 3 + \left(1(3) + \frac{1}{2}(1)(2) + \frac{1}{2}(5)(5) - \frac{1}{2}(1)(2)\right)$$

$$= 3 + \left(3 + \frac{25}{2}\right) = 3 + \frac{31}{2} = \frac{37}{2}$$

b. $\int_{-6}^{-1} g(x)\ dx = \int_{-6}^{-3} g(x)\ dx + \int_{-3}^{-1} g(x)\ dx$

$$= \int_{-6}^{-3} -2(x+4)^2 + 5\ dx + \int_{-3}^{-1} 3\ dx$$

$$= \left[-\frac{2}{3}(x+4)^3 + 5x\right]_{-6}^{-3} + 3x\Big|_{-3}^{-1}$$

$$= -\frac{2}{3}\left[(-3+4)^3 - (-6+4)^3\right] + 5(-3-(-6)) + 3(-1-(-3))$$

$$= -\frac{2}{3}(1+8) + 5(3) + 3(2)$$

$$= -6 + 15 + 6 = 15$$

c. f is decreasing when $f' < 0$ or $g(x) < 0$ and f is concave down when $f'' > 0$ or when the slope of $g(x) < 0$. The graph of f is decreasing and concave down on $6 < x < 7$ because $g(x) < 0$ and $g(x)$ is decreasing on this interval.

d. Points of inflection occur where f' changes from increasing to decreasing, or from decreasing to increasing. There are two inflection points at $x = -4$ and $x = 0$.

4. Solution

a. $T'(9) = \dfrac{T(10) - T(8)}{10 - 8}$

$$= \frac{51 - 60}{10 - 8} = \frac{-9}{2} = -4.5\ C^\circ/\text{cm}$$

b. The average temperature of the wire is $\frac{1}{10}\int_0^{10} T(x)\ dx$. Let $A = \int_0^{10} T(x)\ dx$, then the trapezoidal approximation for A is

$$\frac{100+90}{2}\cdot 2 + \frac{90+81}{2}\cdot 1 + \frac{81+73}{2}\cdot 2 + \frac{73+60}{2}\cdot 3 + \frac{60+51}{2}\cdot 2 = 740$$

The average temperature is $\frac{1}{10}A = \frac{1}{10}\cdot 740 = 74\ C^\circ$.

c. $\int_0^{10} T'(x)\ dx = T(10) - T(0) = 51 - 100 = -49\ C^\circ$

The temperature drops $49\ C^\circ$ from the heated end of the wire to the other end of the wire.

d. $\dfrac{T(5)-T(3)}{5-3}=\dfrac{73-81}{2}=\dfrac{-8}{2}=-4$

Because T is differentiable on $3 \le x \le 5$, T is continuous on $3 \le x \le 5$. By the Mean Value Theorem, there exists a value c, $3 < c < 5$, such that $T'(c)=-4$.

5. Solution:

a. We can rewrite $f(x)$ as

$$f(x)=x^2\cdot\frac{1}{1-(-3x)}$$

Then we can find the first four nonzero terms.

$$x^2\cdot 1+x^2(-3x)+x^2(-3x)^2+x^2(-3x)^3=x^2-3x^3+9x^4-27x^5$$

And the general term is $(-3)^n x^{n+2}$.

b. Use the ratio test.

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\left|\frac{(-3)^{n+1}x^{n+3}}{(-3)^n x^{n+2}}\right|$$

$$=\lim_{n\to\infty}|-3x|=|3x|$$

$|3x|<1$ for $|x|<\dfrac{1}{3}$

Therefore, the radius of convergence of the Maclaurin series for f is $\dfrac{1}{3}$.

When $x=-\frac{1}{3}$, the series is

$$\sum_{n=0}^{\infty}(-3)^n\left(-\frac{1}{3}\right)^{n+2}=\sum_{n=0}^{\infty}\frac{1^n}{9}$$

By the limit test, the series diverges.

When $x=\frac{1}{3}$, the series is

$$\sum_{n=0}^{\infty}(-3)^n\left(\frac{1}{3}\right)^{n+2}=\sum_{n=0}^{\infty}\frac{(-1)^n}{9}$$

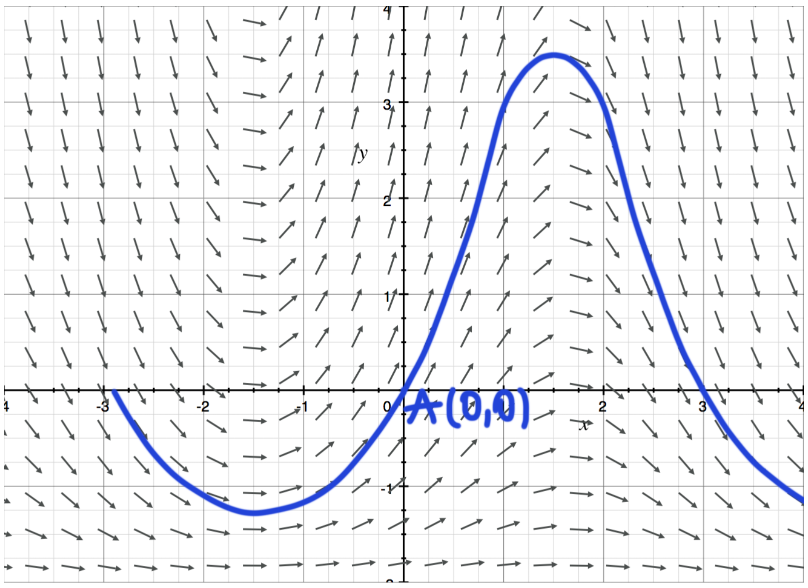
By the geometric series test, the series diverges. The interval of convergence of the Maclaurin series for f is $-\frac{1}{3}<x<\frac{1}{3}$.

c. By the alternating series error bound, the error $\left|P_4\left(\frac{1}{3}\right)-f\left(\frac{1}{3}\right)\right|$ is bounded by the magnitude of the next term of the alternating series.

$$\left|P_4\left(\frac{1}{3}\right)-f\left(\frac{1}{3}\right)\right|<\left|-27\left(\frac{1}{3}\right)^5\right|=\frac{1}{9}<\frac{1}{2}$$

6. Solution:

a. First, we find the point $(0,0)$ on the graph function, and then we draw the curve using the function's slope. The resulting graph is the following:



b. First, we need to evaluate the slope at $(0,0)$.

$$\frac{dy}{dx}(0,0) = (0+2)\cos 0 = 2(1) = 2$$

An equation for the tangent line is $y - y_1 = m(x - x_1)$

Substitute and we get:

$$y - 0 = 2(x - 0)$$

$$y = 2x$$

Therefore

$$f(0.5) = 2(0.5) = 1$$

c. First separate the variables

$$\frac{dy}{dx} = (y+2)\cos x$$

$$\frac{dy}{y+2} = \cos x \ dx$$

Integrate both sides.

$$\int \frac{dy}{y+2} = \int \cos x \ dx$$

$$\ln|y+2| + c_1 = \sin x + c_2$$

$\ln|y+2| = \sin x + c$, where $c = c_2 - c_1$

Substitute the initial condition $f(0) = 0$ and solve for c.

$$\ln|0+2| = \sin 0 + c$$

$$\ln 2 = c$$

$$\ln|y+2| = \sin x + \ln 2$$

Because $y(0) = 0$, $y > -2$, so $|y+2| = y+2$.

$$\ln(y+2) = \sin x + \ln 2$$

$$y + 2 = e^{\sin x + \ln 2}$$

$$y + 2 = 2e^{\sin x}$$

$$y = 2e^{\sin x} - 2$$